FAKIR MOHAN UNIVERSITY
VYASA VIHAR, BALASORE

Curriculum for Three/Four Year Degree Course

(With Multiple Entry /Exit Option)

Based on NEP-2020

Mathematics

Effective for students admitted during 2024-25 and afterwards

Semester	Subjects
I	Paper I - Calculus & Analytic Geometry
	Paper II- Introduction to Algebra & Number Theory
II	Paper III- Real Analysis-I
	Paper IV - Algebra-I
III	Paper V- Probability
	Paper VI- Differential Equations-I
	Paper VII- Linear Algebra
IV	Paper VIII- Real Analysis-II
	Paper IX- Complex Analysis-I
	Paper X- Algebra-II
V	Paper XI- Real Analysis-III
	Paper XII- Differential Equations-II
	Paper XIII- Numerical Analysis & Scientific Computing
VI	Paper XIV- Multivariable Calculus
	Paper XV- Differential Geometry
VII	Paper XVI- Measure Theory & Integration
	Paper XVII- Algebra-III
	Paper XVIII- Topology
	Paper XIX- Mathematical Methods
VIII	Paper XX- Functional Analysis
	Paper XXI- Analytic Number Theory
	Paper XXII- Complex Analysis-II
	Paper XXIII- Differential Equations-III

Programme Outcome

- To prepare the students for a career in Mathematics.
- To prepare the students for Higher Education and Research in Mathematics.
- To develop a conceptual understanding of the subject and to develop an inquisitiveness in the subject.
- To enable the student to acquire basic skills necessary to understand the subject and to master the skills to handle equipment's utilized to learn the subject.
- To generally promote wider reading on the subject and allied inter disciplinary subject.

Paper I

Semester-I

Calculus & Analytic Geometry

Course Objective:

The main emphasis of this course is to equip the student with necessary analytic and technical skills to handle problems of mathematical nature as well as practical problems. More precisely, main target of this course is to explore the different tools for higher order derivatives to plot the various curves and to solve the problems associated with differentiation and integration of vector functions.

Learning Outcomes:

After completing the course the student will be able to

- Trace a curve and find asymptotes.
- Talculate integrals of typical type using reduction formulae, etc.
- Calculate arc length, surface of revolution and know about conics
- Calculate triple products, gradient divergence, curl, etc.

Unit I

Hyperbolic functions, higher order derivatives, Leibnitz rule and its applications to problems of the type $e^{ax+b}sinx$, $e^{ax+b}cosx$, $(ax + b)^n sinx$, $(ax + b)^n cosx$, concavity and inflection points, asymptotes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L'Hospital rule, application in business, economics and life sciences.

Unit II

Riemann integration as a limit of sum, integration by parts, reduction formulae, derivations and illustrations of reduction formulae of the type $\int sin^n xdx$,$\int cos^n xdx$,$\int tan^n xdx$,$\int sec^n xdx$, $\int (logx)^n dx$,$\int sin^n xcos^n xdx$, definite integral, integration by substitution.

Unit III

Volumes by slicing, disks and washers methods, volumes by cylindrical shells, parametric equations, parameterizing a curve, arc length, arc length of parametric curves, area of surface of revolution, techniques of sketching conics, reflection properties of conics, rotation of axes and second degree equations, classification into conics using the

discriminant, polar equations of conics.
Triple product, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation, partial differentiation, div, curl and integration of vector functions, tangent and normal components of acceleration.

LIST OF PRACTICAL

(To be performed using Computer with aid of MATLAS/SCILAB or such software)

1. Plotting the graphs of the functions, $e^{(ax+b)}$, log(ax+b) ,1 /(ax+b), sin(ax+b), cos (ax+b) and |axtb| to illustrate the effect of a and b on the graph.
2. Plotting the graphs of the polynomial of degree 4 and 5.
3. Sketching parametric curves (E.g., Trochoid, cycloid, hypocycloid).
4. Obtaining surface of revolution, of curves.
5. Tracing of conics in Cartesian coordinates/polar coordinates.
6. Sketching ellipsoid, hyperboloid of one and two sheets (using Cartesian coordinates). |.

Books Recommended:

- ✓ *H. Anton, I. Bivens and S. Davis: Calculus,10th Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.*
- ✓ *Shanti Narayan, P. K. Mittal: Differential Calculus, S. Chand, 2014.*
- ✓ *R. J. T Bell: An elementary Treatise on coordinate geometry, MacMillan and Company Limited, 2005.*

Books for Reference:

- ✓ *James Stewart: Single Variable Calculus, Early Transcendental, 8th edition, Cengage Learning, 2016.*
- ✓ *G.B. Thomas and R. L. Finney: Calculus, 9th Ed., Pearson Education, Delhi, 2005.*
- ✓ *M. J. Strauss, G. L. Bradley and K. J. Smith: Calculus, 3rd edition, Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper II

Introduction to Algebra & Number Theory

Course Objectives:

To present a systematic introduction to number theory and a basic course on algebra.

Learning Outcomes:

After completing the course the student will be able to

- Understand the equivalence relations and concept of group with different examples.
- Understand the properties of cyclic groups, rings, and integral domain.
- Know divisibility and division algorithm and find *gcd* using Euclidean Algorithm.
- Solve linear Diophantine equations, find least common multiples,solve linear congruence applying the Chinese remainder theorem.

Unit I

Integers and equivalence relations, properties of integers, modular arithmetic, mathematical inductions, equivalence relations, Introduction to groups, symmetries of a square, the dihedral groups, definitions and examples of groups, elementary properties of groups, subgroups, examples of subgroups.

Unit II

Cyclic groups, properties of cyclic groups, classification of subgroups of cyclic groups, definitions and examples of normal subgroups, Introduction to rings, definition and examples of rings, properties of rings, subrings, definition and examples of integral domain and fields.

Unit III

Divisibility, division algorithms, prime and composite numbers, Fibonacci and Lucas numbers, Fermat numbers, greatest common divisor, Euclidean algorithm.

Unit IV

Fundamental theorem of arithmetic, least common multiple, linear Diophantine equations, congruence, linear congruence, Chinese remainder theorem, Wilson's theorem, Fermat little theorem, Euler's theorem.

Books Recommended:

- ✓ *Joseph A. Gallian, Contemporary Abstract Algebra (4th Edition), Narosa Publishing House, New Delhi, 1999.(IX Edition 2010).*
- ✓ *Thomas Koshy, Elementary Number Theory with Applications (2^{nd} Edition), Academic Press, 2007.*

Books for Reference:

- ✓ *I. N. Herstein: Topics in Algebra, Wiley Eastern Limited, India, 1975.*
- ✓ *David M. Burton: Elementary Number Theory (6th Edition), Tata McGraw-Hill Edition, Indian Reprint, 2007.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org.*

Paper III

Semester II

Real Analysis-I

Course Objective:

The objective of the course is to introduce the basics of real number system and the properties of sequence and series of real numbers. The ideas of completeness, least upper bound property, denseness, limit, continuity and uniform continuity will also be introduced. This is one of the Paper courses essential to start doing mathematics.

Learning Outcomes:

On successful completion of this course, students will be able to

- Learn basics of real number system and test countability of a set.
- Know on sequence of real numbers and their basic properties.
- Test convergence of an infinite series.
- Find limit and continuity of functions and test uniform continuity of functions.

Unit I

Finite and infinite sets, countable and uncountable sets, examples, algebraic and order Properties of R, uncountability of R, completeness property of **R,** applications of the supermom property, Intervals, nested interval property, denseness of rationals in R.

Unit II

Sequence and their limits, limit theorems, monotone sequences, monotone Convergence theorem, subsequences, divergence criteria, monotone subsequence theorem, Bolzano Weierstrass theorem for sequences, Cauchy sequence, Cauchy's convergence criterion.

Unit III

Infinite series, convergence and divergence of infinite series, Cauchy criterion, Tests for convergence: comparison test, limit comparison test, ratio test, Cauchy's nth root test, Raabe's test, integral test, alternating series, Leibniz test, absolute and conditional convergence. Limits of functions, limit theorems, some extensions of limit concept, continuous functions and their combinations, continuous functions on intervals, boundedness theorem, maximum minimum theorem, intermediate value theorem, uniform continuity, examples, uniform continuity theorem.

Unit IV

LIST OF PRACTICALS

(To be performed using Sage Math / Mathematica / Maple / MATLAB / SCILAB etc.)

1. Visualize and compare the behaviour to assess convergence of the harmonic Series $\sum \frac{1}{n}$ and a general power series $\sum \frac{1}{n^p}$
2. Plot the partial sums to visually inspect convergence / divergence of a geometric series and a harmonic series.
3. Check if the series $\sum \frac{1}{n^2}$ satisfies Cauchy's criterion. Visualize the behaviour of the difference between terms for large *n*.
4. Implement Raabe's test for selected series. Visualize the outcome and interpret the region of convergence.
5. Approximate the integral for a series. Plot the function and the integral to see the convergence behaviour. Example : The alternating harmonic series $\sum(-1)^{n+1}\frac{1}{n}$ and the exponential series $\sum \frac{1}{n\,!}$.
6. Apply Leibniz test to the alternating series $\sum(-1)^n \frac{1}{n}$. Plot the partial sums and observe how they approach the limit.
7. Estimate the remainder after a finite number of terms of Taylor Series / Fourier Series for real life applications. Visualize how the error decreases as more terms are summed.
8. Plot a continuous function $f(x)$ on a closed interval $[a, b]$ and verify its boundedness. Example : $f(x) = x^2 - 3x + 5$ in $[-3, 4]$.

Books Recommended:

- ✓ *R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 3rd Edn., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.*
- ✓ *G. Das and S. Pattanayak, Fundamentals of Mathematical Analysis, TMH Publishing Co., 30th reprints, 2021.*

Books for Reference:

- ✓ S. *C. Mallik and S. Arora, Mathematical Analysis, New Age International Publications.*
- ✓ *A. Kumar, S. Kumaresan, A basic course in Real Analysis, CRC Press, 2014.*
- ✓ *Brian S. Thomson, Andrew. M. Bruckner, and Judith B. Bruckner, Elementary Real Analysis, Prentice Hall, 2001.*
- ✓ *Gerald G. Bilodeau , Paul R. Thie, G. E. Keough, An Introduction to Analysis, Jones & Bartlett, Second Edition, 2010.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Suggested digital platform: NPTEL/SWAYAM/MOOCs

Paper IV

Algebra-I

Course Objectives:

To present a systematic and rigorous study on algebraic structures like groups, rings and some important results with their applications. After pursuing this course, one can opt for advanced topics in groups, rings and their applications to
problems in physics, computer science and engineering.

Learning Outcomes:

After completing this course, students will able to

- Understand permutation groups with some results and application in Rubik's cube.
- Understand the concept of homomorphisms, isomorphisms, normal subgroups and factor groups.
- Explore more properties of rings and ideals rigorously.
- Get introduced to the concept of reducibility and irreducibility of polynomials and concept of unique factorization domain.

Unit I

Permutation groups, definition and notations, cyclic notation, properties of permutations, isomorphisms, definition and examples, Cayley's theorem, properties of isomorphisms, automorphisms, cosets, properties of cosets, Lagrange's theorem and consequences, an application of cosets to permutation groups, an application of cosets to Rubik's cube.

Unit II

External direct products, definition and examples, properties of external direct products, the group of units modulo n as an external direct product, applications, normal subgroups, factor groups, application of factor groups, internal direct products, group homomorphisms, definition and examples, properties of homomorphisms, the first isomorphism theorem.

Unit III

Characteristic of a ring, ideals, factor rings, prime ideals and maximal ideals, ring homomorphisms, definition and examples, the field of quotients, polynomial rings, notations and terminology, division algorithm and consequences.

Unit IV

Factorization of polynomials, reducibility test, irreducibility test, unique factorization in Z[x], divisibility in integral domains, irreducible, primes, unique factorization domain, Euclidean domain.

Books Recommended:

- ✓ *Joseph A. Gallian, Contemporary Abstract Algebra (9th Edition), Narosa Publishing House, New Delhi, 2010.*
- ✓ *I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.*

Books for Reference:

- ✓ *John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.*
- ✓ *D. S. Dummit, R. M. Foote, Abstract Algebra, Wiley-India edition, 2013.*
- ✓ *Joseph 1. Rotman, An Introduction to the Theory of Groups, 4th Ed., Springer Verlag, 1995.*
- ✓ *M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper V **Semester III**

Probability

Course Objective:

The objective of the course is to make the student understand basics of probability which is of use in everyday life.

Learning Outcomes:

After completing the course the student will be able to

- Learn the basics of probability and random variables with axioms of probability.
- Know the discrete and continuous distributions and learn how to calculate mean, variance and moments of them.
- Learn on limit theorems with their applications and know about the conditional expectations.
- Learn on Markov chains and their applications.

Unit I

Sample space and events, probability axioms, probability defined on events, conditional probabilities, I ndependent events, Bayes formula, real random variables, discrete and continuous random variables, probability distribution function, probability mass/density functions, mathematical expectation, and properties, variance and standard deviation.

Unit II

Discrete distributions: uniform, binomial, Poisson, geometric, negative binomial, continuous distributions: uniform, normal, exponential, their expectations and variance, moments, moment generating function, characteristic function and computation of these for the distributions, joint distribution function and its properties, joint probability density functions, marginal and conditional distributions, independent random variables.

Unit III

Limit theorems: Markov inequality, Chebyshev's inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers, application to problems, conditional probability and conditional expectation, discrete case, continuous case, applications, expectation of function of two random variables, conditional expectations, bivariate normal distribution, correlation coefficient, joint moment generating function and calculation of covariance, linear regression for two variables.

Unit IV

Central limit theorem for independent and identically distributed random variables with finite variance, Markov chains, Chapman-Kolmogorov equations, classification of states, Gambler Ruin problem.

Book Recommended

1. *Sheldon Ross, Introduction To Probability Models (9th Edition), Academic Press, Indian Reprint, 2007.*
2. *Robert V. Hogg, Joseph W. Mckean And Allen T. Craig, Introduction To Mathematical Statistics, Pearson Education, Asia, 2007.*
3. *Kai Lai Chung, Elementary Probability Theory With Stochastics Process, Springer International Students Edition, (Narosa Publ.)*

Book for Reference

- ✓ *Alexander M. Mood, Franklin A. Graybill and Duane C. Boes, Introduction to the Theory of Statistics, (3rd Edition), Tata McGraw- Hill, Reprint 2007.*
- ✓ *Chow Y S , Teicher H Probability theory Springer International edition*
- ✓ *Irwin Miller and Marylees Miller, John E. Freund's Mathematical Statistics with Applications (7th Edition), Pearson Education, Asia, 2006.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*

Paper VI

Differential Equations-I

Course Objective:

Differential Equations introduced by Leibnitz in 1676 models almost all Physical, Biological, Chemical systems in nature. The objective of this course is to familiarize the students to various methods of solving differential equations, partial differential equations and to have a qualitative application through models. The students have to solve problems to understand the methods.

Learning Outcomes:

After completing the course, the student will be able to

- Get the idea to solve first order linear ordinary differential equations of different types those are arising in physical problems.
- Get the idea to solve second order linear ordinary differential equations of different types those are arising in physical problems.
- Get basic ideas of first order partial differential equations, its formulation in two, three variables and variable separable method for identify the solutions.
- Get idea to solve various mathematical models of ODEs and PDEs which may be helpful for simulation process.

Unit I

Differential equations and mathematical models, general, particular, explicit, implicit and singular solutions of a differential equation, exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equations and Bernoulli's equation, compartmental model, population model for single species.

Unit II

General solution of homogeneous equation of second order, principle of superposition, Wronskian, its properties and applications, method of undetermined coefficients, method of variation of parameters, linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler’s equations.

Unit III

Partial Differential Equations - Basic concepts and definitions, origin of first order PDEs, Classification of first order PDEs, Pfaffian differential forms and equations, solution of Pfaffian differential equations in three variables, Cauchy’s problem for first order PDEs, linear equations of first order, integral surfaces passing through a given curve, Cauchy’s method of characteristics, compatible systems, method of separation of variables for solving first order and second order partial differential equations.

Unit IV (Practical)

The students will implement the following problems in the computer Lab using *Matlab /SCILAB Mathematica / Maple* etc.

1. Plotting of second order solution family of differential equations.
2. Plotting of third order solution family of differentialequations.
3. Population growth model (exponential case only).
4. Population decay model (exponential case only).
5. Solution of Cauchy problem for first order PDEs.
6. Finding the characteristics for the first order PDEs.
7. Plot the integral surfaces of a given first order PDE with initial data.

Books Recommended:

- ✓ *J. Sinha Roy and S. Padhy: A course of Ordinary and Partial differential equations, Kalyani Publishers, New Delhi,2018.*
- ✓ *Belinda Barnes and Glenn R. Fulford, Mathematical Modeling with Case Studies, A Differential Equation Approaching Maple and Matlab, 2nd Edn. Taylor and Francis group, London and New York, 2009.*
- ✓ *Sneddon; Elements of Partial Differential Equations, McGraw-Hill, International Students Edition, 1957.*

Books for Reference:

- ✓ *G. F. Simmons, Differential equation, Tata McGraw Hill, 1991.*
- ✓ *J. N. Sharma and Kehar Singh, PDE for Engineers and Scientists, Narosa, New Delhi, 2009.*
- ✓ *Martin Braun, Differential Equations and their Applications, Springer International Student Ed.1978.*
- ✓ *S. L. Ross, Differential Equations, 3rd Edition, John Wiley and Sons, India, 2014.*
- ✓ *C.Y. Lin, Theory and Examples of Ordinary Differential Equations, World Scientific, 2011.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*

1. e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org

Paper VII

Linear Algebra

Course Objective:

The objective of this course is to acquaint students with matrix operations, solution of system of equations, vector spaces and linear transformations. In addition, the student will learn about eigenvalues, diagonalization, canonical forms, etc., which has many applications in almost all areas of science and engineering.

Learning Outcomes: After completing the course the student will be able to

- Determine basis and the dimension of a finite-dimensional vector space, know the relation between rank and nullity of a linear transformation.
- The relation between matrix and linear transformation.
- To find solution of system of linear equations, compute eigenvalues, eigenvectors of a matrix and linear transformation.
- About orthogonality of vectors and application of it to different form of matrix, introduced to different operators.

Unit I

Vector spaces, subspaces, span of a set, more about subspaces, linear dependence, independence, product and quotient space, dimension and basis, linear transformations, range and kernel of a linear map, rank and nullity of linear map.

Unit II

Inverse of linear transformation, consequences of rank – nullity theorem, the space L(U, V), composition of linear maps, matrix associated with linear map, linear map associated with matrix, rank and nullity of a matrix, determinant minors and rank of a matrix, transpose of a matrix and special type of matrices, elementary row operations

Unit III

System of linear equations, matrix inversion, application of determinant to linear equations, eigenvalues and eigenvectors, similarity of matrices, invariant subspaces, minimal polynomial (eigenvalues and the minimal polynomial), upper triangular matrices, diagonalizable operators (diagonal matrices, conditions for diagonalizability).

Unit IV

Inner product space: inner products and norms, orthonormal bases, orthogonal complements, self-adjoint and normal operators, spectral theorems, isometries, unitary operators, characteristic polynomial, Cayley – Hamilton theorem, Jordan form, trace, quadratic form, application to reduction of quadrics.

Books Recommended:

- ✓ *V. Krishnamurthy. V.P. Mainra, J. L. Arora, An introduction to linear algebra, Affiliated East – West press Pvt. Ltd., New Delhi, 1976.*
- ✓ *Sheldon Axler, Linear algebra done right (Fourth edition),Springer, 2024.*

Books for References:

- ✓ *Seymour Lipschutz and Marc Lars Lipson, Linear Algebra (Schaum's outlines, Fourth Edition), McGraw Hill, New York, 2009.*
- ✓ *A. Ramachandra Rao and P. Bhimsankaram, Linear Algebra (Second Edition), Hindustan Book Agency, 2000.*
- ✓ *Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra (Fourth Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.*
- ✓ *Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007.*
- ✓ *S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India, 1999.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org ; https://linear.axler.net; and https://library.oapen.org/handle/20.500.12657/85067*

Paper VIII **Semester IV**

REAL ANALYSIS-II

Course Objective:

As a second course in real analysis, the objective is to learn on the concept of differentiation, Riemann Integration and their applications. The series of functions and the improper integrals have also been introduced.

Learning Outcomes: After completing the course the student will be able to

- Learn working out problems on derivatives of function and their applications.
- Learn about Riemann Integration and their properties including Improper Integrals.
- Learn on pointwise and uniform convergence of power series.
- Learn to calculate value of improper integrals.

Unit I

Differentiability of a function at a point and in an interval, Caratheodory's theorem, algebra of differentiable functions, relative extrema, interior extremum theorem. Rolle's theorem, Mean value theorems, Cauchy's mean value theorem, Lagrange mean value theorem, intermediate value property of derivatives, Darboux's theorem, applications of mean value theorem, Taylor's theorem and applications.

Unit II

Riemann integration: partitions, conditions of integrability, definition of Riemann integral properties of the Riemann integral, Riemann integral as limit of a sum, mean value theorem for integrals, integration by parts, Fundamental theorems of calculus, Taylor theorem with remainder.

Unit III

Pointwise and uniform convergence of sequence of functions, Cauchy criterion for uniform convergence and Weierstrass M-test, uniform convergence and continuity, term by term integration and differentiation of a series, power series, Abel's theorem, Weierstrass approximation theorem, Taylor series

Unit IV

Improper integrals, integration of unbounded functions with finite limits of integration, comparison tests of convergence, infinite range of integration, integrand as product of functions convergent at infinity, absolutely convergent integral,tests of convergence, convergence of Beta and Gamma functions, applications.

Books Recommended:

- ✓ *R.G. Bartle D.R. Sherbert, Introduction to Real Analysis, John Wiley and Sons (Asia) Pvt. Ltd., Singapore*
- ✓ *G. Das and S. Pattanayak, Fundamentals of Mathematics Analysis, TMH Publishing Co.*
- ✓ *S. C. Mallik and S. Arora, Mathematical Analysis, New Age International Ltd., New Delhi.*

Book for Reference:

- ✓ *K. A. Ross, Elementary Analysis: The theory of Calculus, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.*
- ✓ *Charles G. Denlinger, Elements of Real Analysis, Jones and Bartlett (Student Edition), 2011.*
- ✓ *A. Kumar, S. Kumaresan, A basic course in Real Analysis, CRC Press, 2014*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*

Paper IX

Complex Analysis-I

Course Objectives:

The objective of the course is to introduce the theories for functions of a complex variable. The concepts of analyticity and complex integration and its applications, are discuss in detail. This course is prerequisite to many other advance analysis courses such as advanced complex analysis, geometric functions, theory, potential theory, theory of entire and meromorphic functions, etc.

Learning Outcomes:

After completing the course the student will be able to

- Understand the geometric aspects of complex numbers system, convergence of series of complex numbers.
- Understand the significance of complex differentiability, analyticity and construction of analytic functions from given harmonic functions.
- Relate the notion of line integral, Cauchy fundamental theorems on integrals and its applications.
- Classify the nature of singularities, properties of zeros and poles and be able to know the applications of residue theorem.

Unit I

Basic properties of complex number and, Stereographic projection, power series, absolute convergence, uniform convergence, Cauchy-Hadamard formula for the radius of convergence, circle of convergence, exponential, logarithmic, sine and cosine functions for complex numbers.

Unit II

Continuity and differentiability of a complex valued function, analytic function, necessary and sufficient conditions for analytic functions, Cauchy-Riemann equations (Cartesian and polar form), harmonic and conjugate harmonic functions, construction of analytic function (Milne-Thomson’s method).

Unit III

Line integral, path independence, complex integration, Green’s theorem, anti-derivative theorem, Cauchy-Goursat theorem, Cauchy integral formula, Cauchy's inequality, derivative of analytic function and its generalizations, Liouville's theorem, Morera's theorem, Taylor's and Laurent's theorem, expansion of analytical function in Taylor and Laurent series.

Zeroes of an analytic function, singularities of complex functions and its classifications, residues, Cauchy's residue theorem, residue at infinity, residues at poles and its examples, maximum modulus theorem.

Unit IV

LIST OF PRACTICALS

(To be performed using Sage Math / Mathematica / Maple / MATLAB / SCILAB etc.)

1. Make a geometric plot to show that the n^{th} roots of unity are equally spaced points that lie on the unit circle $C_1(0) = \{z : |z| = 1\}$ and form the vertices of a regular polygon with n sides, for n = 4, 5, 6, 7, 8.

2. Find all the solutions of the equation $z^3 = 8i$ and represent these geometrically.

3. Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units. Show the effect of rotation of this ellipse by an angle of $\frac{\pi}{6}$ radians and shifting of the centre from $(0, 0)$ to $(2, 1)$ by making a parametric plot.

4. Show that the image of the open disk $D_1(-1-i) = \{z: |z+1+i| < 1\}$ under the linear transformation $w = f(z) = (3-4i)\,z + 6 + 2i$ is the open disk: $D_5(-1+3i) = \{w: |w+1-3i| < 5\}$.

5. Show that the image of the right half-plane $Re\, z = x > 1$ under the linear transformation $w = (-1+i)z - 2 + 3i$ is the half -plane $v > u + 7$, where $u = Re(w)$, etc. Plot the map.

6. Show that the image at the right hall-plane $A = \{z: Re\, z \geq \frac{1}{2}\}$ under the mapping $w = f(z) = \frac{1}{z}$ is the closed disk $D_1(1) = \{w: |w-1| \leq 1\}$ in the w-plane.

7. Make a plot of the vertical lines $x = a$, for $a = -1, -\frac{1}{2}, \frac{1}{2}, 1$ and the horizontal lines $y = b$, for $b = -1, -\frac{1}{2}, \frac{1}{2}, 1$. Find the plot of this grid under the mapping $f(z) = \frac{1}{z}$.

8. Find a parameterization of the polygonal path $C = C_1 + C_2 + C_3$, from $-1 + i$ to $3 - 4i$ where C_1 is the line from $-1 + i$ to -1, C_2 is the line from -1 to $1 + i$ and C_3 is the line from $1 + i$ to $3 - i$. Make a plot of this path.

9. Plot the line segment $'L'$ joining the point $A = 0$ to $B = 2 + \frac{\pi}{4} i$ and give an exact calculation of $\int_L e^z\, dz$.

10. Evaluate $\int_C \frac{1}{z-2}\, dz$, where C is the upper semicircle with radius 1 centered at $z = 2$ oriented in a positive direction.

Books Recommended:

- ✓ *Elias M. Stein & Rami Shakarchi: Complex Analysis, Princeton University press,*

Princeton and Oxford, 2003.

- ✓ *Joseph Bak and Donald J. Newman: Complex analysis (3rd Edition), Undergraduate Texts in Mathematics, Springer-Verlag, NewYork, 1997.*

Books for Reference:

- ✓ *S. Ponnusamy and Herb Silverman: Complex variables with Applications: Birkhauser, (2006) (Indian Edition 2012).*
- ✓ *H. A. Priestly: Introduction to Complex Analysis, Oxford University Press, 2008.*
- ✓ *Donald Sarason: Complex Function Theory: AMS, Second Edition, 2007.*
- ✓ *James Ward Brown and Ruel V.Churchill: Complex Variables and Applications (Eighth Edition), McGraw-Hill International Edition, 2009.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper X

Algebra-II

Course Objectives:

To present a systematic study on finite abelian groups, Sylow's theorems and Modules.

Learning Outcomes:

After completing the course, the student will be able to

- Know on finite abelian groups, the class equation and Sylow's theorems.
- Know on applications of Sylow's theorems and test the simplicity of groups.
- Learn on group action, composition series, nilpotent groups and solvable groups.
- Solve problems in modules and related results.

Unit I

Fundamental theorem of finite abelian groups, isomorphism classes of abelian groups, proof of the fundamental theorem, Sylow's theorems, conjugacy classes, the class equation, Sylow's first theorem, Cauchy theorem, Sylow's second and third theorems.

Unit II

Application of Sylow's theorem, finite simple groups, non-simplicity tests, the simplicity of alternating group A_5, free groups, classification of groups of order up to 15, characterization of dihedral groups.

Unit III

Group actions and permutation representations, composition series and holder programs, nilpotent groups, solvable groups.

Unit IV

Introduction to modules, definition and examples, direct sum, free modules, quotient modules, homomorphisms, simple modules, modules over PIDs.

Books Recommended:

- ✓ *Joseph A. Gallian, Contemporary Abstract Algebra (4th Edition), Narosa Publishing House, New Delhi, 1999.(IX Edition 2010).*
- ✓ *D. S. Dummit, R. M. Foote, Abstract Algebra, Wiley-India edition, 2013.*
- ✓ *C. Musili, Introduction to Rings and Modules, Narosa Publishing House.*

Books for Reference:

- ✓ *John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.*
- ✓ *N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.*
- ✓ *M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.*
- ✓ *S. Nanda, Topics in Algebra, Allied Publishers, New Delhi.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in; http://ocw.mit.edu; http://mathforum.org*

Paper XI **Semester V**

Real Analysis-III

Course Objective:

After a first course in real analysis in undergraduate program, the ideas of uniform continuity, uniform convergence and approximation by polynomials are crucial in analysis. In addition to the functions of bounded variation and their integrators, the student has to learn differentiating functions from R^n to R^m. The techniques of integration of a function with respect to another function and the basic ideas of finding a Fourier series are also included.

Learning Outcomes:

After completing the course the student will be able to

- Find the Fourier series of a function.
- Calculate Riemann Stieltjes integrals and know whether a function is of bounded variation or not.
- Learn how to define derivatives on R^n including the existence of partial derivatives, inverse function theorem and implicit function theorem.
- Learn about metric spaces and their topological properties.

Unit I

Basic concepts of Fourier series, Fourier series of even and odd functions, half range series, Fourier series on other intervals, orthogonal systems of functions, theorem on best approximation, properties of Fourier coefficients, Riesz-Fisher theorem, Riemann-Lebesgue lemma, Dirichlet integral, Integral representation for the partial sum of a Fourier series**,** convergence of Fourier series.

Unit II

Function of bounded variation, examples, total variation, function of bounded variation expressed as difference of increasing functions, rectifiable paths, Riemann-Stieltjes integrals, properties and techniques, necessary and sufficient condition for existence of the integral, mean value theorem for Riemann-Stieltjes integrals, reduction to Riemann integrals.

Unit III

Differentiation in R^n, partial derivatives, directional derivatives, sufficient condition for differentiability, chain rule, , mean value theorem, Jacobians, contraction mapping principle, inverse function theorem, implicit function theorem, rank theorem, differentiation of integrals, Taylor theorem in many variables.

Unit IV

Metric spaces, definitions and examples, open and closed sets, interior and exterior points, convergence and completeness, continuity and uniform continuity, compactness, connectedness.

Books Recommended:

- ✓ *W. Rudin, Principles of Mathematical Analysis, McGraw Hill, 3rd edition.*
- ✓ *T. Apostol, Mathematical Analysis, Pearson, 2nd edition.*
- ✓ *S C Malik and Savita Arora, Mathematical Analysis- New Age International, 5th edition*

Books for Reference:

- ✓ *Terrence Tao, Analysis-I, Hindustan book agency.*
- ✓ *Terrence Tao, Analysis-II, Hindustan book agency*
- ✓ *K. A. Ross, Elementary Analysis: The theory of Calculus, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.*
- ✓ *Charles G. Denlinger, Elements of Real Analysis, Jones and Bartlett, Student Edition, 2011.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XII

Differential Equations-II

Course Objective:

The objective of this course is to understand basic methods for solving nonlinear first order ordinary differential equations and existence of solutions along with some special type of second order ordinary differential equations of mathematical physics. Also, students will be exposed to second order partial differential equations arising in thermal physics and thermodynamics.

Learning Outcomes:

After completing the course the student will be able to

- Understand first order nonlinear ordinary differential equations and existence of solutions
- Learn the methods to find solutions of second order linear ordinary differential equations with constant coefficients and variable coefficients.
- The different methods for solving first and second order partial differential equations and can take more courses on wave equation, heat equation, diffusion equation, gas dynamics, nonlinear evolution equations etc. All these courses are important in engineering and industrial applications for solving boundary value problems.
- Get idea to solve various mathematical models of ODE and PDE which may be helpful for simulation process.

Unit I

Existence and Uniqueness of Solutions: Lipschitz condition, Gronwall type inequality, successive approximations, Picard's theorem, non-uniqueness of solutions, continuation and dependence on initial conditions, existence of solutions in the large.

Unit II

Solution of second order ODE with constant coefficients, power series solutions of ordinary and singular points, and special functions of Legendre's differential equations, Bessel's differential equations and their properties.

Unit III

Charpit's method, special types of first order PDE, Jacobi's method, Linear second order PDE, canonical forms of second order PDE and characteristics curves, one dimensional wave equation, its origin and elementary solutions, vibration of an infinite string, vibration of a semi finite string, vibration of a string of finite length, existence of unique solution.

Unit IV (Practical)

Laboratory work for the following problems using *MATLAB /SCILAB /Mathematica / Maple* etc.

1) Plot the Fourier series of the following functions:
   i. $f(x) = x^2, x \in [-1, 1]$
   ii. $f(x) = \begin{cases} 1, & 0 < x < \pi \\ -1, & -\pi < x < 0 \end{cases}$
   iii. $f(x) = \sin x, \ \ 0 < x < \frac{\pi}{2}$
2) Solution of wave equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ for the following associated conditions:
   (i) $u(x,0) = \varphi(x), u_t(x,0) = \sigma(x), x \in \mathbb{R}, t > 0$
   (ii) $u(x,0) = \varphi(x), u_t(x,0) = \sigma(x), u(0,t) = 0 \ x \in (0,\infty), t > 0$
   (iii) $u(x,0) = \varphi(x), u_t(x,0) = \sigma(x), u_x(0,t) = 0 \ x \in (0,\infty), t > 0$
   (iv) $u(x,0) = \varphi(x), u_t(x,0) = \sigma(x), u(0,t) = 0, \ u(l,t) = 0, \ \ 0 < x < l, t > 0$
3) Solution of one dimensional heat equation $\frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = 0$ for the following conditions
   (i) $u(x,0) = \varphi(x), u(0,t) = a, u(l,t) = b, 0 < x < l, t > 0$
   (ii) $u(x,0) = \varphi(x), x \in \mathbb{R}, 0 < t < T$
   (iii) $u(x,0) = \varphi(x), u(0,t) = a, x \in (0,\infty), t \geq 0.$

Books Recommended:

- ✓ *Deo and Raghavendra, Text Book of Ordinary Differential Equations, Tata McGraw-Hill Pub. Company Ltd, New Delhi,2017.*
- ✓ *Simmons G F, Differential equation, Tata McGrawHill,1991.(Ch-5. 28-30, Ch-8. 44-47, Ch-6, 33-36)*
- ✓ *I. Sneddon, Elements of Partial Differential Equations,McGraw-Hill, International Students Edition.(Ch-2. 10,11,13; Ch-3. 4-7; Ch-5. 1,2), 2006.*
- ✓ *Tyn Myint-U and Lokenath Debnath, Linear Partial Differential Equations for Scientists and Engineers, 4th edition, Birkhauser, Indian reprint,2014.*

Books for Reference:

- ✓ *J. N. Sharma and Kehar Singh, PDE for Engineers and Scientists, Narosa, New Delhi,2009.*
- ✓ *T Amarnath, An Elementary Course in Partial Differential Equations, Narosa Publications, 2003.*
- ✓ *Martin Braun, Differential Equations and their Applications, Springer International Student Ed. 1978.*
- ✓ *S. L. Ross, Differential Equations, 3rd Edition, John Wiley and Sons, India, 2014.*
- ✓ *C.Y. Lin, Theory and Examples of Ordinary Differential Equations, World Scientific, 2011.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ;* http://mathforum.org

- *Suggested digital platform: NPTEL/SWAYAM/MOOCs*

Paper XIII

Numerical Analysis & Scientific Computing

Course Objectives:

The objective of this course is to acquaint the students with a wide range of numerical methods to solve algebraic and transcendental equations, linear system of equations, interpolation and curve fitting problems, numerical integration, initial and boundary value problems, etc. Develop adequate skills to apply those methods in real world problems.

Learning Outcomes:

After completing the course the student will be able to

- Understand the errors in computation, find the roots of algebraic and transcendental equations, familiarize with convergence, advantages and limitations of those numerical techniques, learn to apply Gauss−Jacobi, Gauss−Seidel methods to solve system of linear equations.
- Get aware of using interpolation techniques to solve polynomials.
- Learn numerical differentiation and integrations by using different techniques.
- Understand the techniques to find approximate solutions of ODE and PDE.

Unit I

Errors in approximation, absolute, relative and percentage errors, round-off error, solution of algebraic and transcendental equations: bisection method, Regula-Falsi method, secant method, method of iteration, Newton Raphson method, order of convergence, systems of simultaneous equations: Gauss elimination method, Gauss Jordon method, LU decomposition method, Iterative methods: Jacobi method and Gauss-Seidel method.

Unit II

Finite differences, interpolation techniques for equal intervals-Newton forward and backward, Gauss forward, Gauss backward, interpolation, interpolation with unequal intervals-Newton's divided difference method, Lagrange method, Hermite interpolation, Numerical differentiation using Newton forward and backward formulae, numerical integration using Newton-Cotes formulas, trapezoidal rule, Simpson rules, Gauss-Legendre, Gauss-Chebyshev formulas.

Unit III

Solution of ordinary differential equations: Taylor series method, Picard's method, Euler method, Euler modified method, Runge–Kutta methods.

Unit IV (Practical)

Practical / Lab work to be perform in Computer Lab:
Use of computer algebra system (CAS) software: Python/ Sage Math / Mathematica/ MATLAB/ Maple/ Maxima/ Scilab/ R or any other (open source) software etc., for developing at least the following numerical programs:

1. Bisection method, Newton-Raphson method and Secant method.
2. LU decomposition method.
3. Gauss−Jacobi method and Gauss−Seidel method.
4. Lagrange interpolation and Newton interpolation.
5. Trapezoidal rule and Simpson's rules.
6. Taylor series method, Picard's method, Euler method, Euler modified method and Runge−Kutta Methods.

Note: ***Non-programmable scientific calculator is allowed in the examination.***

Books Recommended:

- ✓ *M. K. Jain, S. R. K. Iyengar & R. K. Jain: Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, India, 2016.*
- ✓ *R. K. Gupta: Numerical Methods: Fundamentals and Applications, Cambridge University Press, 2019.*

Books for Reference:

- ✓ *Brian Bradie: A Friendly Introduction to Numerical Analysis. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third impression, 2011.*
- ✓ *Curtis F. Gerald & Patrick O. Wheatley: Applied Numerical Analysis, Pearson Education. India, 2007.*
- ✓ *S. D. Conte & S. de Boor: Elementary Numerical Analysis: An Algorithmic Approach, 1980.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XIV **Semester VI**

Multivariable Calculus

Course Objectives:

The primary objective of this course is to introduce students, the extension of the studies of single variable differential and integral calculus to functions of two or more independent variables with the geometry and visualization of curves and surfaces. To aware the students about the techniques multiple integrations and higher order derivatives.

Learning Outcomes:

After completing the course the student will be able to

- Learn the concept of limit, continuity and differentiations of functions of more than one.
- Understand the maximization and minimization of multivariable functions with the given constraints on variables.
- Learn about inter-relationship amongst the line integral, double, and triple integral formulations.
- Familiarize with the Green's, Stokes' and Gauss divergence theorems and their applications.

Unit I

Functions of several variables, limit and continuity of functions of two variables: partial differentiation, total differentiability, sufficient condition for differentiability, chain rule for one and two independent parameters, directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes.

Unit II

Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems, double integration over rectangular region, double integration over non rectangular region, double integrals in polar co-ordinates.

Unit III

Triple integrals, triple integral over a parallelepiped and solid regions, volume by triple integrals, cylindrical and spherical co- ordinates, change of variables in double integrals and triple integrals. Definition of vector field, divergence and curl, line integrals, applications of line integrals: mass and work, fundamental theorem for line integrals, conservative vector fields, independence of path, Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stokes' theorem, the divergence theorem

Unit IV

LIST OF PRACTICALS

(To be performed using Sage Math / Mathematica / Maple / MATLAB / SCILAB etc.)

1. Plot the gradient vector field on scalar functions such as temperature distribution scalar field.
2. Compute and plot the tangent plane to a surface defined by multivariable function at a point.
3. Apply the Lagrange multiplier method to real-world optimization problems like maximizing area given a perimeter constraint. Plot the constraint curve and solution points. Example: Maximize $f(x,y) = xy$ subject to $x^2 + y^2 = 1$.
4. Calculate $\int_0^1 \int_0^2 xy\, dxdy$ using both symbolic and numerical methods. Plot the region of integration.
5. Evaluate $\int_0^1 \int_0^{1-x} (x+y)\, dxdy$ using numerical integration. Use 3D plots to visualize the integration region and surface.
6. Evaluate $\int_0^{2\pi} \int_0^1 r\, drd\theta$ (area of a unit circle). Plot the region of integration and verify results using both Cartesian and polar co-ordinates.
7. Use numerical integration to compute and plot the volume of a parallelepiped.
8. Define and plot the boundaries of a solid region (e.g., sphere, cone, cylinder etc.). Evaluate the volume by setting up and computing the triple integral.
9. Compute and plot the volumes of solid regions like spheres and spherical caps using spherical coordinates.
10. Evaluate & plot the volume of a cone using cylindrical coordinates. The cone is described by $z = \sqrt{x^2 + y^2}$ with a height of $z = 1$.

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Books Recommended:

- ✓ *M. J. Strauss, G. L. Bradley and K. J. Smith: Calculus, 3rd Edition, Dorling Kindersley (India) Pvt. Ltd. Pearson Education, Delhi, 2007.*
- ✓ *E. Marsden, A. J. Tromba and A. Weinstein: Basic Multivariable Calculus, Springer Student International Edition, Indian reprint, 2005.*

Book for References:

- ✓ *S. C. Mallik and S. Arora: Mathematical Analysis, New Age International Publications, New Delhi, 2005.*
- ✓ *Tom Apostol: Mathematical Analysis, Narosa Publishing House, 2002.*
- ✓ *G. B. Thomas and R. L. Finney: Calculus, 9th Ed., Pearson Education, Delhi, 2005.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XV

Differential Geometry

Course Objective:

The objective of this course is to explore geometry of curves and surfaces in R^2 and R^3 with their intrinsic properties and curvatures.

Learning Outcomes:

After completing the course the student will be able to

- Understand the notion of plane curves, space curves, curvature, torsion and the existence of space curves.
- Learn the theory of sur faces and learn to calculate first fundamental forms.
- Learns on geodesics on a surface and learns to calculate curvatures.
- Learns calculating second fundamental forms, curvatures and discovers minimal surfaces.

Unit I

Theory of Space Curves: space curves, arc length, tangent, normal and binormal, osculating plane, curvature, torsion, Serret-Frenet formulae, contact between curves and surfaces, osculating circles and spheres, involute and evolutes, existence of space curves, Helices.

Unit II

Theory of surfaces: parametric curves on surfaces, surfaces of revolution, helicoids, metric, direction coefficients. First Fundamental forms.

Unit III

Geodesics, canonical geodesic equations, nature of geodesics on a surface of revolution, normal property of geodesics, Torsion of a geodesic: geodesic curvature, Gauss-Bonnet theorem, Gaussian curvature, surfaces of constant curvature.

Unit IV

Second Fundamental forms, principal curvatures, lines of curvature, Rodrigue’s formula, conjugate and asymptotic lines. Developables, developable associated with space curves and curves on surfaces, minimal surfaces. Fundamental Theory of surfaces.

Books Recommended:

- ✓ *T.J. Willmore, An Introduction to Differential Geometry, Dover Publications, 2012.*
- ✓ *Pressley, Elementary Differential Geometry, Springer International Edition, 2014*

Books for References:

- ✓ *O'Neill, Elementary Differential Geometry, 2nd Ed., Academic Press, 2006.*
- ✓ *C.E. Weatherburn, Differential Geometry of Three Dimensions, Cambridge University Press 2003.*
- ✓ *D.J. Struik, Lectureson Classical Differential Geometry, Dover Publications, 1988.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XVI **Semester VII**

Measure Theory & Integration

Course Objectives:

The aim of this course is to provide a foundation for student to many branches of mathematics such as functional analysis, harmonic analysis, ergodic theory, and probability theory, etc.. In this course, the students will be introduced to, Lebesgue measure and integration, signed measures, Hahn-Jordan decomposition, Radon-Nikodym derivative and product measures.

Learning Outcomes:

After completing the course the student will be able to

- Calculate Riemann-Stieltjes integrals and Lebesgue integrals of simple measurable functions.
- Know how to calculate Lebesgue Integral of any measurable functions and learn how to apply monotone and dominated convergence theorems.
- Learn the concept of measure on abstract spaces and work on various modes of convergence of a sequence of measurable functions.
- Learn on complex measures, Radon Nikodym derivatives and related results.

Unit I

Lebesgue outer measure, measurable sets, Borel sets, regularity, measurable functions, Borel and Lebesgue measurability, non-measurable sets, integration of nonnegative functions, simple functions, Lebesgue integration of simple function.

Unit II

Approximation of measurable functions by simple functions, Lebesgue integral of measurable functions and properties, Fatou's lemma, monotone convergence theorem, Lebesgue dominated convergence theorem, integration of series, Riemann and Lebesgue integrals, differentiation, Dini derivatives, Lebesgue differentiation theorem.

Unit III

Abstract measure spaces, measure and outer measure, extension of a measure, uniqueness of the extension, completion of a measure, integration with respect to a measure, Modes of convergence, convergence in measure, almost uniform convergence, fundamental in measure convergence, Egorov's theorem.

Unit IV

Signed measure, absolute continuity, Hahn decompositions, Jordan decomposition, Lebesgue decomposition, Radon-Nikodym theorem, applications of Radon Nikodym Theorem, product measure, Fubini theorem.

Books Recommended:

- ✓ *G. De Barra: Measure Theory and Integration, New age International, 1981.*
- ✓ *H. L. Royden: Real Analysis, Pearson, Fourth Edition, 2010.*

Books for Reference:

- ✓ *H. L. Royden and P. M. Fitzpatrick: Real Analysis, Fourth Edition, Pearson Asia Education Ltd and China Machine Press, 2010.*
- ✓ *C. D. Aliprantis, O. Burkinshaw: Principles of Real Analysis, Elsevier, 2011.*
- ✓ *J . Yeh: Real Analysis (Theory of Measure and Integration), 3rd Edition, World Scientific Publication, 2024.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XVII

Algebra-III

Course Objectives:

To present a systematic study of field theory and Galois theory.

Learning Outcomes:

After completing the course the student will be able to

- Understand the basic concept of field extension, and splitting fields.
- Understand the significance of separable extension, cyclotomic polynomials, Galois group.
- Understand the structures and properties of finite fields, composite extensions, simple extensions.
- Determine the Galois group of a polynomial and understand the conditions under which polynomial equations can be solved using radicals.

Unit I

Basic theory of field extension, algebraic extension, classical straightedge and compass construction, splitting fields and algebraic closures.

Unit II

Separable and inseparable extension, Cyclotomic polynomials and extensions, Galois theory, basic definitions, The fundamental theorem of Galois theory.

Unit III

Finite fields, Composite extensions and simple extensions, cyclotomic extensions and abelian extensions over Q, Galois groups of polynomials over Q.

Unit IV

Solvability and radical extension, Insolvability of quintic, computations of Galois group over Q.

Books Recommended:

- ✓ *D. S. Dummit, R. M. Foote, Abstract Algebra, Wiley-India edition, 2013.*
- ✓ *Joseph A. Gallian, Contemporary Abstract Algebra (4th Edition), Narosa Publishing House, New Delhi, 1999.(IX Edition 2010).*

Books for Reference:

- ✓ *John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.*
- ✓ *I.N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.*

- ✓ *e-Learning Source http://ndl.iitkgp.ac.in; http://ocw.mit.edu; http://mathforum.org*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*

Paper XVIII

Topology

Course Objective:

This is an introductory course in Topology. The objective of this course is to have knowledge on topological spaces, continuity, connectedness, compactness and separation axioms. Topology on quotient spaces, product spaces and metric spaces are also discussed. The student will also learn on basic ideas of algebraic topology such as homotopy, fundamental groups and covering spaces.

Learning Outcomes:

After taking the course the student will be able to

- Know on basics of topological spaces with examples and is able to construct new topologies using idea of product topology, quotient topology, etc .
- Solve problems involving continuous maps, homeomorphisms between two spaces , connectedness and compactness.
- Learn examples and properties of Hausdorff, regular, normal, separable, first and second countable spaces.
- Understand more results in separation axioms and learn on basic concepts of algebraic topology like homotopy, fundamental groups, and covering spaces.

Unit I

Cartesian product of a family of sets, Axiom of choice and its equivalents(without proof),Topological spaces, examples, open sets, closed sets, basis and subbasis for a topology, closure and interior of sets, subspace topology, order topology, continuous functions, homeomorphisms, product topology, quotient topology.

Unit II

Metric topology, standard topology, uniform topology, lower limit topology, connectedness, examples, local connectedness, Path-connectedness, connected subsets of real line, compact spaces, examples, locally compacts spaces, sequential compactness, limit point compactness, compact subsets of real line.

Unit III

Countability axioms, first and second countable spaces, separable and Lindolf spaces, separation axioms, regular & completely regular space, normal spaces, Urysohn Lemma.

Unit IV

Urysohn metrization theorem, Tychonoff theorem, compactness in metric spaces, compact open topology homotopy of paths, fundamental group, covering space.

Books Recommended:

- ✓ *J R Munkres, Topology: A First Course, Pearson, 2nd edition, 2000.*
- ✓ *M A Armstrong, Basic Topology. Springer, 1983.*

Books for Reference:

- ✓ *K. D. Joshi, Introduction to General Topology, Wiley Eastern Limited.*
- ✓ *T. S. Singh, Elements of Topology, CRC press (special Indian Edition) 2015.*
- ✓ *O. Viro, O Ivanov, V Kharlamov and N Netsvetaev, Elementary Topology, problem Text book, American Mathematical society, 2008.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XIX

Mathematical Methods

Course Objective:

The objective of this course is to prepare a student in basics of Integral transforms, Integral equations and calculus of variations. These tools have applications in other science and engineering fields and are necessary to understand.

Learning Outcomes:

After completing the course the student will be able to

- Calculate Laplace transform, Fourier transform and apply them in areas of differential equations immediately.
- **F**ind solutions of Volterra integral equations.
- Find solutions of Fredholm integral equations.
- Use methods in calculus of variations to solve extremal problems in Differential equations and physics.

Unit I

Laplace transforms: definitions, properties, Laplace transforms of some elementary functions, convolution theorem, inverse Laplace transformation and applications. Fourier transforms: definitions, properties, Fourier transforms of some elementary functions, convolution, Fourier transform of derivatives, Fourier transforms as a limit of Fourier series.

Unit II

Volterra Integral Equations: Basic concepts, relationship between linear differential equations and Volterra integral equations, resolvent kernel of Volterra integral equations, solution of integral equations by resolvent kernel, The method of successive approximations, convolution type equations, solutions of integral differential equations with the aid of Laplace transforms.

Unit III

Fredholm integral equations: Fredholm equations of the second kind, fundamentals, iterated kernel, constructing the resolvent kernel with the aid of iterated kernels, integral equations with degenerate kernels, characteristic numbers and eigen functions, solution of homogeneous integral equations with degenerate kernel, non-homogeneous symmetric equations, Fredholm alternatives.

Unit IV

Calculus of variations: extremal of functional, The variation of a functional and its properties, Euler's equations, field of extremals, sufficient conditions for the extremum of a functional, conditional extremum moving boundary problem, discontinuous problems, one sided variations, Ritz method.

Books Recommended:

- ✓ *A. J. Jerri; Introduction to Integral Equations with Applications, John-Wiley & SONS, INC., 1999.*
- ✓ *Lokenath Debnath; Integral Transforms and Their Applications, CRC Press, New York.*
- ✓ *A. S. Gupta; Calculus of Variations with Applications, PHI, Pvt. Ltd., New Delhi.*

Books for Reference:

- ✓ *I. Sneddon, The use of Integral Transformations (Tata McGraw Hill), 1972.*
- ✓ *Murray R Spiegel, Laplace Transforms, Schaum's Series, 1965.*
- ✓ *Gelfand and Fomin, Calculus of Variations, Dover Pub, 2003.*
- ✓ *Krasnov, Problems and Exercises in Calculus of Variations, Mir Publ., 1970.*
- ✓ *Ram P Kanwa, Linear Integral Equations (Academic Press), 2013.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XX **Semester VIII**

Functional Analysis

Course Objective:

The objective of this course is to introduce students to L^p spaces, Banach Spaces, Hilbert Spaces etc. Students will also be exposed to bounded linear operators on Hilbert spaces which is required to study quantum mechanics, scattering theory and spectral theory, etc. Knowledge of real Analysis, measure theory and linear algebra is pre requisite for this course

Learning Outcomes:

After completing the course the student will be able to

- Handle inequalities in L^p spaces, and normed linear spaces.
- Learn all basic results on Hilbert spaces for further application.
- Know on Fourier series with respect to an orthonormal basis and related results and basic results of Banach space
- Know on more results on Banach spaces and bounded linear operators with spectrum on Banach spaces.

Unit I

Review of Metric spaces (not a part of examination), L^p and l^p spaces, inequalities (Holder, Minkowski, Jensen), completeness of Lp, denseness and separability, normed linear spaces, properties of normed linear spaces, continuity of linear maps.

Unit –II

Inner product spaces, examples, Hilbert spaces, examples, closed subspaces, existence of a unique element of smallest norm, orthogonal complements and properties, projection theorem, Riesz representation theorem, orthonormal sets, Gram-Schimdt orthonormalization.

Unit – III

Orthonormal basis, Fourier expansion, Bessel's inequality, Riesz-Fischer theorem, Parseval's formula, Banach spaces, examples, Hahn Banach theorem, Baire's category theorem.

Unit IV

Open mapping theorem, closed graph theorem, uniform boundedness principle, duals of $L^p[a, b]$, bounded linear operators on Banach spaces, spectrum of a bounded operators and properties, resolvent set and examples.

Books Recommended:

- ✓ *B. V. Limaye -Functional Analysis, 3rd Ed, 2014.*
- ✓ *E. Kreyszig-Functional Analysis –Wiley-India, 2007.*

Books for Reference

- ✓ Goffman and Pedrick A first Course in Functional Analysis, AMS, 2017.
- ✓ J. B. Conway, A course in Functional Analysis, 2nd Ed., Springer, 2006
- ✓ P. K. Jain and O. P. Ahuja, Functional Analysis, 2nd Ed., New Age International Publication, New Delhi, 2004
- ✓ Markus Haase, Functional Analysis: An Elementary Introduction, American Mathematical Society, 2014.
- ✓ Suggested digital platform: NPTEL/SWAYAM/MOOCs.
- ✓ e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org

Paper XXI

Analytic Number Theory

Course Objectives:

The aim of this course is to study number theory by using analytic tools such as inequalities, limits, calculus, etc.

Learning Outcomes:

After completing the course the student will be able to

- Understand the arithmetical functions and their relations.
- Find the average order of multiplicative functions and know the distribution of prime numbers.
- Know the prime number theorem and Ramanujan's sum
- Know the basic theory of Riemann zeta function and related L-function.

Unit I

The arithmetical functions and their relations, Mobius function, Euler totient function, Mangolt function, Liouville's function, The, divisor function, The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, multiplicative functions. The Bell series of an arithmetical function and Dirichlet multiplications, Derivatives of arithmetical functions, The Selberg identity.

Unit II

The big oh notation, Euler's summation formula, some elementary asymptotic formulas, averages of arithmetical functions, The average order of divisor functions, The average order of Euler totient function, The average order of Mobious and Mangoldt functions, The partial sums of a Dirichlet product, applications to the Mobius and Mangoldt functions, some elementary theorems on distribution of prime numbers, Chebyshev's functions and their relations with $\pi(x)$.

Unit III

Some equivalent forms of the prime number theorem, Shapiro Tauberian theorem, The partial sums of the Mobius function, brief sketch of an elementary proof of the prime number theorem, Ramanujan's sum and generalizations, quadratic residues, Legendre's symbol and its properties, Gauss's lemma, The quadratic reciprocity law, The Jacobi symbol**.**

Unit IV

The half-plane of absolute convergence of a Dirichlet series, Euler products, analytic properties of Dirichlet series, mean value formulas for Dirichlet series, an integral formulas for the coefficients and the partial sums of a Dirichlet series, The Riemann zeta function and the L-function, properties of the gamma function, integral representation for the Hurwitz zeta function, analytic continuation of the Hurwitz zeta function, analytic continuation of the Riemann zeta function and the L-function.

Books Recommended:

- ✓ *T. M. Apostol, Introduction to Analytic Number Theory, Springer International Edition, 2010.*
- ✓ *Analytic Number Theory: Exploring the Anatomy of Integers, Jean-Marie De Koninck, Florian Luca, American Mathematical Society, 2012.*

Books For Reference:

- ✓ *A Primer of Analytic Number Theory: From Pythagoras to Riemann, Jeffrey Stopple, Cambridge University Press, 2003.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XXII

Complex Analysis-II

Course Objectives:

This course introduces the basic concepts of conformal mappings, entire functions, Weierstrass infinite products, Hadamard's factorization theorem, Gamma function, Zeta function and normal family.

Learning Outcomes: On the completion of this course, students will be able to

- Solve problems involving conformal mappings.
- Understand the applications of Cauchy integrals and properties of harmonic functions.
- Handle Gamma function, Riemann zeta function and familiarize with analytic continuations.
- Solve problems involving infinite products, equicontinuity and normal family.

Unit I

Mappings of elementary functions and cross ratio, bilinear transformations and its properties, mapping of some elementary functions, mappings of z^2, e^z, sin z, log z, z+1/z, etc., conformal mappings.

Unit II

Maximum modulus theorems, Schwartz lemma, Argument principle, Rouche's theorem, applications to fundamental theorem of calculus, uniqueness and identity theorems, Hurwitz's theorem, Harmonic functions, mean value theorem, Poisson integral formula, Harnack's inequality and theorem, Hadamard three circle theorem.

Unit III

Weierstrass' factorization theorem, Gamma function and its properties, Riemann zeta function, Riemann's functional equation, Mittag-Leffler's expansion theorem and its applications, analytic continuation, uniqueness of direct analytic continuation, uniqueness of analytic continuation.

Unit IV

Canonical products, Jensen's formula, Poisson-Jensen formula, Hadamard's three circles theorem, order of an entire function, exponent of convergence, Borel's theorem, Hadamard's factorization theorem. equicontinuity, normal family, families of analytic functions.

Books Recommended:

- ✓ *L. V. Ahlfors: Complex Analysis: McGraw Hill, 3rd Edition (2017).*
- ✓ *S. Ponnusamy and Herb Silverman: Complex variables with Applications: Birkhauser, (2006) (Indian Edition 2012).*

Books for References:

- ✓ *J. Bak and D. J. Newman: Complex analysis (3rd Edition), Undergraduate Texts in Mathematics, Springer-Verlag, NewYork, 1997.*
- ✓ *H. A. Priestly: Introduction to complex analysis, Oxford University Press, 2008.*
- ✓ *D. Sarason: Complex Function Theory: AMS, Second Edition, 2007.*
- ✓ *E. M. Stein and R. Shakarchi: Complex analysis: Princeton University Press, 41 William Street, Princeton, New Jersey, 2003.*
- ✓ *John B. Conway: Function of one complex variable: Springer International Student Edition, Narosa Publishing House, Second Edition, 2002.*
- ✓ *R. V. Churchill, J. W. Brown and R. F. Verhey: Complex variables and applications, McGraw Hill, 9th Edition, 2013.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

Paper XXIII

Differential Equations-III

Course Objective:

The objective of this course is to

- Understand the basic methods for qualitative behavior of solutions of ordinary differential equations and boundary value problems,
- Understand the basic methods to solve system of differential equations,
- Expose students about some of the real life problems using the system of differential equations,
- Expose the students to canonical forms of hyperbolic, elliptic and parabolic PDEs.

Learning Outcomes:

On the completion of this course, students will be able to

- Handle oscillation properties of ordinary differential equations and sturm Liouville differential equations
- Know existence and uniqueness theorems and application to mathematical modelling.
- Learn solutions of heat equation and various boundary value problems for Laplace equation
- Solve various PDEs using Green's function

Unit I

- **Oscillation of Second Order Linear Differential Equations:** Fundamental results, Sturm's Comparison Theorem and Hille-Wintner type oscillation.
- **Second Order Boundary Value Problem:** Sturm-Liouville differential equation, eigen value problems, Green's function and Picard's Theorem.

Unit II

System of first order equations, existence and uniqueness theorems, fundamental matrix, homogeneous and nonhomogeneous linear systems with constant coefficient, mathematical formulation of Predatory-pray model, epidemic model of influenza, battle model and their solutions.

Unit III

One dimensional heat equation and its origin, Heat conduction problem for an infinite rod and finite rod, existence and uniqueness of solution, two dimensional heat equation and Laplace equation, boundary value problems, maximum and minimum principles, uniqueness and continuity theorems, Dirichlet problem for a circle, Dirichlet problem for annulus, Neumann problem for a circle.

Unit IV

Solution of heat equation, wave equation, Laplace equation and Helmholz equation by Green's function method and examples.

Books Recommended:

- ✓ *Deo and Raghavendra; Text Book of Ordinary Differential Equations, Tata McGraw-Hill Pub. Company Ltd, New Delhi, 2017.*
- ✓ *Belinda Barnes and Glenn R. Fulford; Mathematical Modeling with Case Studies, A Differential Equation Approaching Maple and Matlab, 2nd Ed., Taylor and Francis group, London and NewYork,2009.*
- ✓ *Tyn Myint-U and Lokenath Debnath; Linear Partial Differential Equations for Scientists and Engineers, 4th edition, Birkhauser, Indian reprint,2014.*

Books for References:

- ✓ *K. Shankar Rao, Introduction to partial differential equations, PHI learning private Ltd., 2011.*
- ✓ *J. N. Sharma, K. Singh, Partial Differential Equations for Engineers and Scienists, Narosa, 2nd Edition, 2009.*
- ✓ *Robert C.McOwen: Partial Differential Equations, Pearson Education Inc., 2002.*
- ✓ *Martin Braun, Differential Equations and their Applications, Springer International Student Ed., 1978.*
- ✓ *S.L. Ross, Differential equations,3rd Ed., John Wiley and Sons, India, 2014.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*

THRUST AREAS FOR DISSERTATION/PROJECT WORK

The student should work for his dissertation in a topic in one of these areas or any area related to these:

1. Topics in Number theory and its applications to Cryptography, Coding theory, etc.
2. Topics in recent development in Group theory, Ring and Field theory
3. Special topics in Number theory like Ramanujan Sum, Distribution of primes
4. Fibonacci and Lucas numbers, Riemann Zeta function, etc.
5. Linear algebra, Matrix theory and applications.
6. Representation theory.
7. Topics in Ordinary Differential Equations such as Stability theory, Oscillation theory.
8. Bifurcation and Catastrophe theory.
9. Finite element method.
10. Initial and boundary value problems in Ordinary and Partial differential equation.
11. Difference equations.
12. Discrete dynamical system such as Julia sets, Horse shoes, Cellular Automata. etc., with topological properties and applications.
13. Fractional calculus.
14. Complex analysis.
15. Fractals and their applications.
16. Geometric function theory including univalent (harmonic) mappings, Quasi conformal maps in one and several variables.
17. Orthogonal polynomials in **R** and **C**, *q*-Calculus and Moments problem.
18. P-adic analysis and P-adic fields.
19. Measure theory.
20. Functional analysis and applications.
21. Summability theory.
22. Geometry of Banach spaces.
23. Ergodic theory.
24. Information theory.
25. Fourier analysis.
26. Wavelets.
27. Fixed point theory.
28. General Topology.
29. Combinatorial Topology.
30. Differential Topology.
31. Algebraic topology including Homology, Homotopy theory, Fundamental groups, Covering spaces, CW complexes, etc.
32. Algebraic Geometry and related topics.
33. Normed algebras.
34. Topics in Operator theory and Operator algebras.
35. Operators on function Spaces including Composition operators, Toeplitz operators, Hankel operators, etc.
36. Spectral theory for bounded and unbounded operator on Hilbert space including Scattering theory.
37. Mathematical modeling (Pollution modeling, Epidemic modeling, Population modeling, etc.).
38. Mathematical Biology including Mathematics of life sciences.

39. Differentiable manifolds, differential forms and related topics.
40. Lie Groups and Lie algebras.
41. Knots and Braids.
42. Relativity, Cosmology and Gravitation, etc.
43. Nonlinear programing problems and applications.
44. Queuing theory.
45. Optimization and Combinatorial optimization.
46. Topics in Artificial Intelligence, Machine learning, Manifold learning and Principal component analysis, etc.
47. Data structure and Data base management, etc.
48. Fuzzy theory and Rough sets.
49. Topics in applications of Probability, Stochastic models including Markov Process, Brownian process, Birth and Death processes, Renewal and Branching, and Reliability, etc.
50. Time series and Forecasting.
51. Topics in Integrable Models such as KDV and KP equation , Toda Lattice, Veselov Novikov, Davey Stewartson and Benney equations, etc.
52. Topics in Image Processing, Signal processing, discrete Image processing, etc.
53. Topics in Graph theory and Combinatorics.
54. Topics in Numerical analysis, Numerical solution for ODE and PDE, etc.
55. Time scale calculus.
56. Computational fluid dynamics.
57. Projective geometry.
58. Finite Field